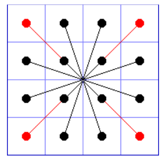
Theorem 10 [connection figure (1234 5678 8765 4321)]



(i) Let k,r,s,N be natural numbers, such that the 16 numbers

(*) 1, k+1, r+1, s+1, k+r+1, k+s+1, r+s+1, k+r+s+1,
N-k-r-s, N-r-s, N-k-s, N-k-r, N-s, N-r, N-k, N

are pairwise different and positive.
Then (*) is a symmetric subset of {1,...,N}, and there are (at least) 64 general 4x4 magic squares M01,...,M64 with entries from (*) and connection figure (1234 5678 8765 4321), namely:

M01 =

1	k+r+s+1	N-r	N-k-s
N-s	N-k-r	r+s+1	k+1
N-k	N-r-s	k+r+1	s+1 ,
k+s+1	r+1	N-k-r-s	N

M33 =

k+1	r+s+1	N-k-r	N-s
N-k-s	N-r	k+r+s+1	1
N	N-k-r-s	r+1	k+s+1,
s+1	k+r+1	N-r-s	N-k

(Note, that M33 is the image of M01, using the map //1234 4321 5678 8765//, i.e.: entries belonging to the same number were exchanged.)
M02,...,M32 are the images of M01 using the 31 transformations of general 4x4 magic squares, which are not the identity, and M34,...,M64 are the images of M33, using the same 31 transformations.

(ii) Let T be a symmetric subset of {1,...,N}, containing the element 1 and let M be a general 4x4 magic square with entries from T and connection figure (1234 5678 8765 4321).
Then there exists a triple (k,r,s) of natural numbers, such that T consists of the 16 numbers (*).
If 1 is a diagonal element of M then M is one of the 32 squares M01,…,M32; otherwise M is one of the 32 squares M33,...,M64.
Let z be the number of triples (k,r,s), such that T consists of the numbers(*), then z=6 and there are exactly 64*6=384 general 4x4 magic squares with entries from T.

Proof

(i) can be verified easily.
Using only linear algebra, the first part of (ii) is shown by solving the linear equations involved.
Moreover z=6, because, if (k,r,s) is a generating triple for T, then the triples (k,s,r), (r,k,s), (r,s,k), (s,r,k), and (s,k,r) generate T, too.
Obviously there are no two different triples (k,r,s) with k<r<s such that T Consists of the 16 numbers (*).

Remark

Let M* be the square M01 of Theorem 1.1, then the above square M01 of Theorem 10 is the image of M* under the transformation:

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     c01 c02 c03 c04          c01 c16 c11 c06
     c05 c06 c07 c08          c13 c04 c07 c10
F:   c09 c10 c11 c12   -->    c09 c08 c03 c14.
     c13 c14 c15 c16          c05 c12 c15 c02
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F is a one to one mapping from the set of general 4x4 magic squares of connection figure (1122 3344 5566 7788) to the set of general 4x4 magic squares of connection figure (1234 5678 8765 4321).