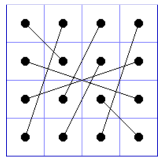
Theorem 9 [connection figure (1234 5167 7385 2648)]



(i) Let k,r be natural numbers with either

(A) r < k < 1.5r, (B) 1.5r < k < 2r, (C) 2r < k < 2.5r,
(D) 2.5r < k < 3r (E) 3r < k < 4r, (F) 4r < k < 5r,
(G) 5r < k < 6r, (H) 6r < k.

Then the 16 numbers

(*) 1,r+1,2r+1,3r+1,4r+1,5r+1,k+1-r,k+1+2r,
k+1+3r,k+1+6r,2k+1,2k+1+r,2k+1+2r,2k+1+3r,2k+1+4r,2k+1+5r

are pairwise different and (*) is a symmetric subset of {1,...,N}, N=2k+1+5r, with difference vector, casewise:

(A) k-r,2r-k,r,2(k-r),3r-2k,k-r,3r-2k,k-r,k-r,3r-2k,2(k-r),r,2r-k,k-r

(B) k-r,2r-k,r,r,2k-3r,2(r-k),2r-k,2k-3r,2r-k,2r-k,2k-3r,r,r,2r-k,k-r

(C) r,k-2r,3r-k,r,r,k-2r,k-2r,5r-2k,k-2r,k-2r,r,r,3r-k,k-2r,r

(D) r,k-2r,3r-k,r,r,k-2r,3r-k,2k-5r,3r-k,2r-k,r,r,3r-k,k-2r,r

(E) r,r,k-3r,4r-k,r,r,k-3r,r,k-3r,r,r,4r-k,k-3r,r,r

(F) r,r,r,k-4r,5r-k,r,k-3r,r,k-3r,r,5r-k,k-4r,r,r,r

(G) r,r,r,r,k-5r,6r-k,k-3r,r,k-3r,6r-k,k-5r,r,r,r,r

(H) r,r,r,r,r,k-6r,3r,r,3r,k-6r,r,r,r,r,r.

Moreover, there are 4 different general 4x4 magic squares M1, M2 , M3, M4 with entries from (*) and connection figure (1234 5167 7385 2648), namely

	5r+1	2r+1	2k+1+r	2k+1+2r
	k+1+3r	2k+1	r+1	k+1+6r
M1 =	k+1-r	4r+1	2k+1+5r	k+1+2r
	2k+1+3r	2k+1+4r	3r+1	1.

M2 is derived from M1 by interchange of 1 and 2k+1,
M3 and M4, are obtained from M1 and M2 by a 180 degree rotation.

(ii) Let N be a natural number of form N=7r+1, 2<r and let k be a natural number with either

(I) 2 < k < r+2, (J) k < r+1 < 2k-1, then the 16 numbers

(**) 1,3r+1,4r+1,7r+1,k,r+k,2r+k,3r+k,
4r+k,5r+k,2r+2-k,3r+2-k,4r+2-k,5r+2-k,6r+2-k,7r+2-k

are pairwise different and (**) is a symmetric subset of {1,...,N} with difference vector, casewise:

(I) k-1,r,r+2-2k,2k-2,r+2-2k,k-1,k-1,r+2-2k,k-1,k-1,r+2-2k,2k-2,r+2-2k,r,k-1

(J) k-1,2r+2-2k,2k-2-r,2r+2-2k,2k-2-r,r-k+1,r-k+1,2k-2-r,r-k+1,r+1-k,2k-2-r,
2r+2-2k,2k-2-r,2r+2-2k,k-1

Moreover, there are 4 different general 4x4 magic squares M5, M6, M7, M8 with entries from (**) and connection figure (1234 5167 7385 2648), namely

	5r+k	2r+k	3r+2-k	4r+1-k
	4r+1	2r+2-k	k+r	7r+1
M5 =	1	4r+k	7r+2-k	3r+1
	5r+2-k	6r+2-k	k+3r	k

$$M6 = \begin{matrix} 7r+2-k & 4r+2-k & k+r & k+2r \\ 4r+1 & k & 3r+2-k & 7r+1 \\ 1 & 6r+2-k & k+5r & 3r+1 \\ k+3r & k+4r & 5r+2-k & 2r+2-k \end{matrix}$$

M7 and M8 are obtained from M5 and M6 by a 180 degree rotation.

(iii) Every general magic square M of connection figure (1234 5167 7385 2648) with entries from a symmetric subset of {1,...,N}, containing the number 1, is of the form either M1,M2,M3,M4,M5,M6,M7,or M8, and the corresponding difference set of this subset is of the form either A,B,C,D,E,F,G,H,I, or J.

Proof (i) and (ii) can be verified by a simple calculation,
(iii) can be proved by solving the linear equations for M.

Remark

(i) For each symmetric subset of {1,...,N} containing the number 1, and allowing a general 4x4 magic square of connection figure (1234 5167 7385 2648) there are exactly 4 different general 4x4 magic squares with entries from this subset.

(ii) There is an imbedding map i from the set of general 4x4 magic squares with connection figure (1234 5167 7385 2648) into the set of general magic squares of connection figure (1122 3443 5665 7788). The map i is defined by:

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    c01 c02 c03 c04       c01 c06 c15 c04
    c05 c06 c07 c08       c05 c14 c07 c12
i:  c09 c10 c11 c12  -->  c09 c10 c03 c08.
    c13 c14 c15 c16       c13 c02 c11 c16
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This follows from the additional equation c01+c03+c14+c16=2(N+1), valid for every general 4x4 magic square of connection figure (1234 5167 7385 2648).