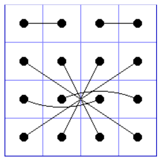
<u>Theorem 6</u> [connection figure (1122 3456 7878 6543)]



(i) Let k,r be natural numbers with either

(A) $1 < k < r+1$, (B) $r+1 < k < 2r+1$,
(C) $2r+1 < k < 3r+1$, (D) $3r+1 < k < 5r+1$
(E) $5r+1 < k < 6r+1$, (F) $6r+1 < k < 7r+1$,
(G) $7r+1 < k < 8r+1$, (H) $8r+1 < k$.

Then the 16 numbers

(*) $1, r+1, 2r+1, 3r+1, 5r+1, 6r+1, 7r+1, 8r+1,$
$k, k+r, k+2r, k+3r, k+5r, k+6r, k+7r, k+8r$

are pairwise different and (*) is a symmetric subset of $\{1,...,N\}$, $N=k+8r$, with difference vector, casewise:

(A) $k-1, r-k+1, k-1, r-k+1, k-1, r-k+1, k-1, 2r-k+1, k-1, r-k+1, k-1, r-k+1, k-1, r-k+1, k-1$

(B) $r, k-r-1, 2r-k+1, k-r-1, 2r-k+1, k-r-1, r, 2r-k+1, r, k-r-1, 2r-k+1, k-r-1, 2r-k+1, k-r-1, r$

(C) $r, r, k-2r-1, 3r-k+1, k-2r-1, r, 3r-k+1, k-2r-1, 3r-k+1, r, k-2r-1, 3r-k+1, k-2r-1, r, r$

(D) $r, r, r, k-3r-1, r, 4r-k+1, k-3r-1, 4r-k+1, k-3r-1, 4r-k+1, r, k-3r-1, r, r, r$

(E) $r, r, r, 2r, k-5r-1, 6r-k+1, k-5r-1, 6r-k+1, k-5r-1, 6r-k+1, k-5r-1, 2r, r, r, r$

(F) $r, r, r, 2r, r, k-6r-1, 7r-k+1, k-6r-1, 7r-k+1, k-6r-1, r, 2r, r, r, r$

(G) $r, r, r, 2r, r, r, k-7r-1, 8r-k+1, k-7r-1, r, r, 2r, r, r, r$

(H) $r, r, r, 2r, r, r, r, k-8r-1, r, r, r, 2r, r, r, r$.

Moreover, there are 4 different general 4x4 magic squares M1, M2, M3, M4 with entries from (*) and connection figure (1122 3456 7878 6543), namely

$$M1 = \begin{matrix} k+6r & 2r+1 & 6r+1 & k+2r \\ k+r & 3r+1 & 5r+1 & k+7r \\ 8r+1 & k+8r & k & 1 \\ r+1 & k+3r & k+5r & 7r+1, \end{matrix}$$

$$M2 = \begin{matrix} 6r+1 & k+2r & k+6r & 2r+1 \\ r+1 & k+3r & k+5r & 7r+1 \\ k+8r & 8r+1 & 1 & k \\ k+r & 3r+1 & 5r+1 & k+7r. \end{matrix}$$

M3 and M4, are obtained from M1 and M2 by reflection at a vertical axis.

(ii) Every general magic square M with entries from a symmetric subset $\{1,...,N\}$, with connection figure (1122 3456 7878 6543), and with entry 1, is of the form either M1,M2,M3, or M4, and the corresponding difference vector of this subset is of the form either A,B,C,D,E,F,G, or H.

<u>Proof</u>

(i) can be verified by a simple calculation,
(ii) can be proved by solving the linear equations for M.

<u>Remark</u>

For each symmetric subset of $\{1,...,N\}$ containing the number 1, and allowing a general 4x4 magic square of connection figure (1122 3456 7878 6543) there are exactly 4 different general 4x4 magic squares with entries from this subset.
There exists an imbedding from the set of general 4x4 magic squares of connection figure (1122 3456 7878 6543) into the set of general 4x4 magic squares of connection figure (1234 2567 8653 4871) [see Theorem 11, Remark 2 (i)].